

# Clinical and Computational Study of Geometry & Hemodynamics of Arterial Stenosis

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**Abstract**— Stenosis is abnormal narrowing of blood vessels. The presence of stenosis in arteries may cause critical flow conditions. It may finally lead to stroke and heart-attack. A clinical study has been done on more than 130 patients along with computational study using 2D axisymmetric rigid model of stenosis in the carotid artery. Assumed shapes of deposition zone and the degree of occlusion used in the analysis were taken from clinical data. The Navier-Stokes equations for incompressible fluid flow have been considered as the governing equations and it has been solved with varying flow parameters using standard CFD software package. The radial velocity profiles at various points of the flow field, the centerline velocity plot and the centerline pressure plots have been obtained from computational study and compared with the clinical data.

**Index Terms**— Arterial flow, Clinical validation, Computational Fluid Dynamics, Hemodynamics, Mathematical Modeling, Stenosis, Stenosis geometry.

## 1 INTRODUCTION

IN the present century arterial stenosis is one of the major causes behind death in all parts of the globe.

Arterial Stenosis is abnormal narrowing or restriction present in the inner wall of blood vessel due to the deposition of cholesterol, fatty materials, cellular waste etc. It may happen in all large or small arteries, commonly in Coronary artery, Carotid artery, and Peripheral artery. In the present case study we only emphasize on Carotid artery stenosis. Carotid artery is one of the larger arteries, present in our neck region. The normal geometry of the artery is divided into three segments, Common Carotid Artery (CCA), External Carotid Artery (ECA) & Internal Carotid Artery (ICA). The deposition of plaque may vary in shape: simple to complex structures and in dimension. Flow through these complex structures is commonly associated with flow separation, stagnation, recirculation, secondary vortex motion, plaque rupture etc.

Efforts have been made to model stenosis and its complex hemodynamics by Computational Fluid Dynamics (CFD) and experimental analyses since 1990's. Ku and others have made detailed studies on the fluid mechanics of vascular systems hemodynamic changes due to stenoses [1, 2]. Johnston and Kilpatrick (1991) studied the effect of geometrical irregularities in the wall of a stenosed artery for Reynolds numbers from 20 to 1000 [3]. Tang et al [1995-1998] used 3D models for steady viscous flow in an

elastic stenotic tube with various stenosis stiffness and pressure conditions [4]. In past experiments blood flow has been considered both as Newtonian or Non-Newtonian fluid depending upon the radius of the blood vessel. Hemodynamic studies have been made for both steady and pulsatile flows. However, no special emphasis is given in the stenosis geometry and previous studies used idealized models using definite curves (Cosine curve, Smooth curves, Irregular Geometry). In this study, a detail care has been taken to obtain more realistic stenosis geometry after going through more than 130 patient's real time Ultrasound Doppler Examination data.

## 2 CLINICAL STUDY

### 2.1 Data Collection

More than 130 ultrasound images of vascular stenosis have been acquired for our analysis of Carotid arterial stenosis. All these patient data have been collected randomly from different well-known multi specialty hospitals in eastern India. It has been ensured during data collection and throughout the work that no patient identity is revealed. Only the information about age and sex has been noted along with other necessary clinical information for every patient data.

### 2.2 Study and Analysis

Each patient data are reviewed thoroughly and very carefully to identify the common geometry and occurrence of stenosis/plaque in the artery. Maximum and minimum deposition heights, length of constriction, percentage of diametric reduction are also noted for each and every data.

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### 3 COMPUTATIONAL STUDY

#### 3.1 Physical Model

In the following study, an axis-symmetric geometry has been developed by considering the carotid artery to be a long straight pipe with radius  $R_0 = D_0/2$  and length  $L_0 = 875R_0$ , where  $D_0$  is taken as 5.7mm (validated from different medical books).

The statistical analysis of the 130 patients reveals two dominant geometries with varied dimensions:

1. Curved shape (Fig 1)
2. Rectangular shape (Fig 2)

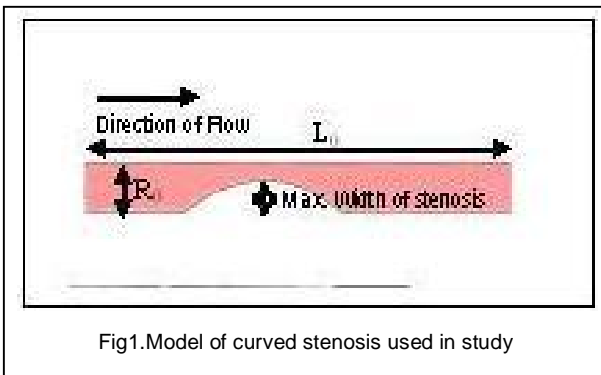


Fig1. Model of curved stenosis used in study

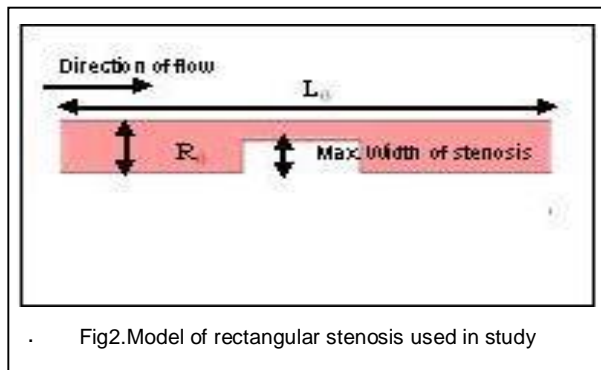


Fig2. Model of rectangular stenosis used in study

The occurrence of the deposition shows three dominant patterns

1. Single sided deposition
2. Axis-symmetric deposition
3. Non axis-symmetric deposition

All of them are considered with a maximum diametric constriction of 62% which can be specified as a moderate degree of stenosis, as a constriction of less than 50% is considered mild and above 70% is considered severe in most medical literature.

Sufficient length of the artery downstream of the stenosis has been taken so that the blood coming out of the constricted region is fully developed at the outlet of the artery. The upstream length for all of the stenosis is considered at  $Z=0.031$ .

#### 3.2 Mathematical Model

##### 3.2.1 Governing equations

The blood flow can be considered to be Newtonian when flowing through large arteries [1, 12]. In our study, as the common carotid artery has been dealt with the flow is considered Newtonian, laminar, steady-state and incompressible.

The incompressible Navier-Stokes Equations along with the continuity equation have been used as the governing equation for modeling the fluid flow.

$$\rho(\mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \cdot [-p\mathbf{I} + \nu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)] \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (2)$$

Where  $u$  is the axial velocity,  $p$  is the axial pressure,  $\nu$  is the dynamic viscosity and  $\rho$  is the density of blood and  $T$  is the transpose matrix. Equation (1) is the momentum balance equation and equation (2) is the continuity equation. In the current study the density of blood has been considered as 1050 Kg/m<sup>3</sup> and the dynamic viscosity as 0.00345 Pa.s.

##### 3.2.2 Boundary conditions

The imposed boundary conditions are:

1. A fully developed velocity profile at the inlet. The equation of the velocity profile is parabolic as expected in laminar flow

$$u(r) = \bar{u} \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \quad (3)$$

where,

$u(r)$  = radial velocity at an arbitrary radius

$\bar{u}$  = mean velocity

$R$  = the radius of the artery

$r$  = the radius at which the velocity is to be obtained.

2. A zero pressure with no viscous stress condition at the outlet.
3. A no-slip condition at all the walls.  
i.e.  $u = 0$

#### 3.3 Numerical Procedure

Standard Finite Element CFD based software COMSOL® 3.5a has been used for the solution of the problems. The solver type is parametric and the solver used is Direct (PARDISO).

#### 3.4 Mesh Details and Grid Sensitivity Test

A free mesh consisting of triangular elements has been used in the study with the maximum possible refinement. The mesh has been refined in the vicinity of the constric-

tions so as to present a more accurate picture of their effects on the blood flow. In the curved constriction, 14900 elements and for the rectangular constriction 15369 elements have been used for solution of the problems.

Grid sensitivity tests for all the simulations have been performed. For all of the stenosis geometries, there have been no noticeable changes in results when the grids have been refined above the values mentioned. So the above refinement of meshes is used in our subsequent studies.

### 3.5 Code Validation

In the absence of any standardized data regarding the haemodynamics of stenosed arteries, results of flow through a straight pipe without any constriction has validated the code. The plug fluid flow considered at the inlet is fully developed after a certain distance from it. The radial velocity profile of the fully developed flow is parabolic and the maximum velocity is the centerline velocity and its value is twice the mean velocity. All these results are fully compliant with the known results of classical fluid mechanics.

## 4 RESULTS

### 4.1 Clinical Results

Both of the carotid arteries have been viewed starting from CCA to ICA and ECA the sites of deposition is as follows:

TABLE1  
AREA OF OCCURRENCE OF PLAQUE

Deposition of plaque along the artery	Occurrence
Common Carotid Artery (CCA)	36.15%
Internal Carotid Artery (ICA)	30.7%
External Carotid Artery (ECA)	12.3%
Bifurcation / Bulb region	20%

In this 2D longitudinal and cross sectional ultra sound images plaque is visible in either side as well as both (inner and outer) sides of the inner vessel wall. ["Outer side" means upper wall boundary of the 2D ultrasound image. Clinically it is towards periphery of the neck region and the "Inner side" is the lower wall boundary of the 2D ultrasound image. Clinically it is away from the periphery of the neck region]. This type of both-sided plaque formation shows axis symmetric and non-axis symmetric formations from single to multiple appearances.

Now, when we examine the 2D B-mode (black and white mode) longitudinal section images for both side depositions more carefully, it shows maximum 18.51% data with very small almost, non-measurable one sided deposition, where other side contributes for a good degree of diametric reduction. Calcification is prominent in 25% cases for the over all batch. Cross sectional images

indicate symmetric and asymmetric deposition.

TABLE2  
APPEARANCE OF PLAQUE

Appearance	Occurrence
Single sided Deposition	54.04%
Both Sided Deposition	45.96%
Multiple Deposition	19%(Both side), 1.4% (single side)
Outer wall Deposition	14.92%
Inner wall Deposition	85.07%

Four common shapes are seen which can be broadly categorized as Cosine shaped, Rectangular shaped, conical shape and spherical or elliptical shaped. Apart from this few irregular geometries are also observed. The common trend is towards the Cosine shaped geometry (Fig.3) and rectangular geometry (Fig.4), but any possible combination of abovementioned geometry is noticed for the deposition in both walls.

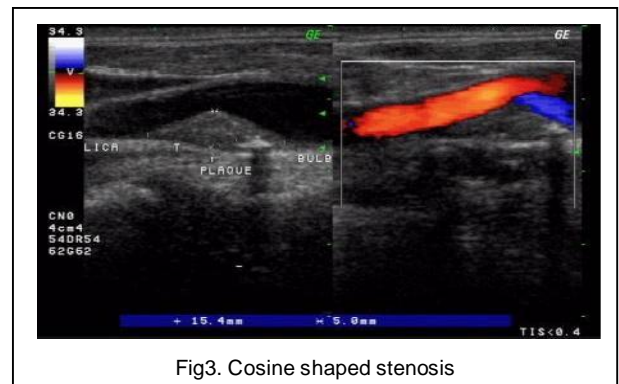


Fig3. Cosine shaped stenosis

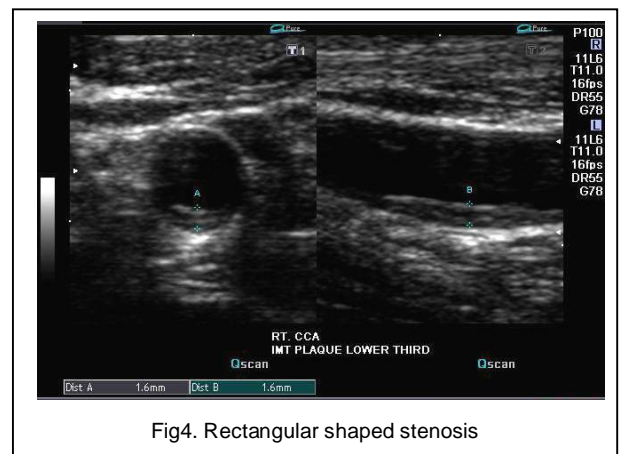


Fig4. Rectangular shaped stenosis

Size variation is very much prominent through out the batch. Large size plaques are present along with multiple small size plaques. For the both (inner and outer) sided deposition a comparison in length and width of geometry

is provided in Fig 5 and Fig 6.

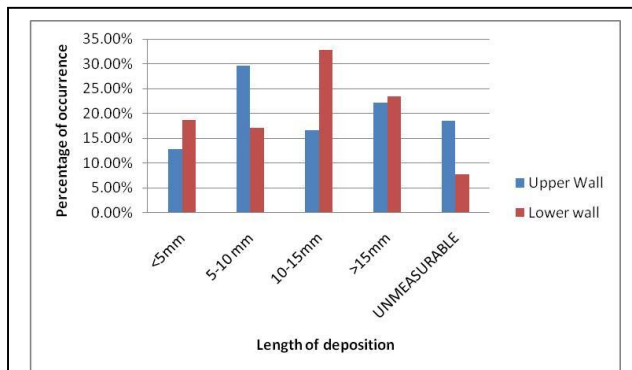


Fig5. Comparison of length of deposition in both walls

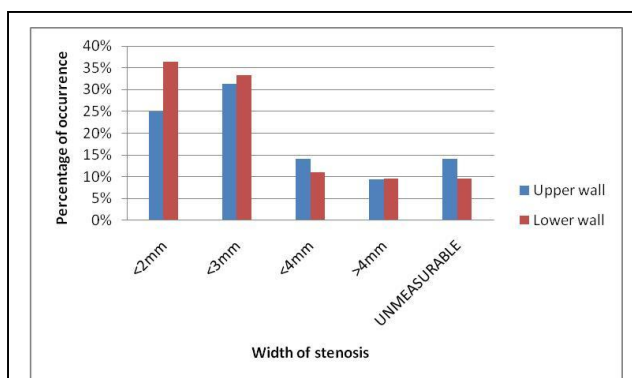


Fig6. Comparison of width of deposition in both walls

From the Fig 5, it is clear that in all the cases of small (<5mm), large (10-15mm) and very large (>15mm) deposition, inner side plaque dimension is much more than outer side. Even in case of maximum width of Deposition the combined effect of both inner and outer wall maximum width provides a high degree of diametric reduction.

More stenosis is found in Male patients as compared to Females falling in same age group, though the sample volume of the clinical study was not so large. Data shows a chance of stenosis is more for person crossing the age of 50-70 years depending on their foodhabbit, life sty and past medical records.

## 4.2 Computational Results

In the available literatures, blood has been found to flow with Reynolds Number (Re) between 100 to 1000. So in this study, for both the stenosis geometries, the flow has been studied with Re = 100, 400, 800 and 1000. A zone of recirculation and an irreversible pressure rise have been observed at the outlet of the stenosis for both rectangular and cosine model. The following points have been observed by studying the simulated results of the rectangular and curved stenoses.

### 4.2.1 Centerline pressure plot

From the centerline pressure plot, it has been observed that at the inlet of the stenosis the pressure fall is higher for higher values of Reynolds Number (Re). Even negative pressure values have been found in case of Re = 1000. But at the outlet of the stenosis, the flows with higher Re show higher values of pressure. So, the irreversible pressure rise increases with increasing values of Re. (Fig. 7)

When comparing the pressure profiles of rectangular and curved stenoses at a fixed Reynolds Number, the irreversible pressure rise has been found to be higher in case of the rectangular geometry. At Re = 1000, the pressure rise for the rectangular stenosis is found to be 23% higher than the curved geometry. (Fig. 8)

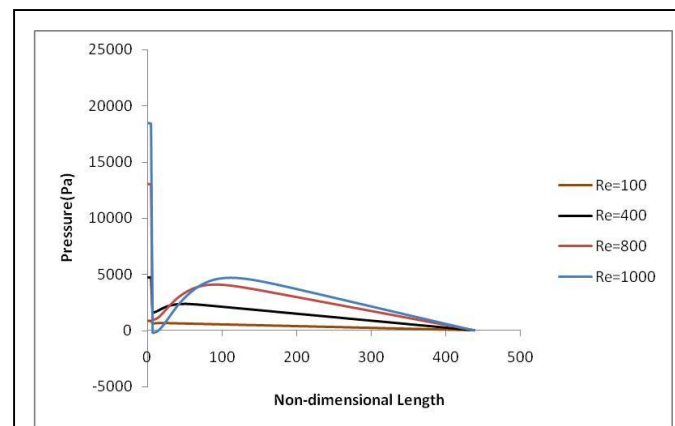


Fig7. The centerline pressure plots of a curved stenosis at different Reynolds Numbers

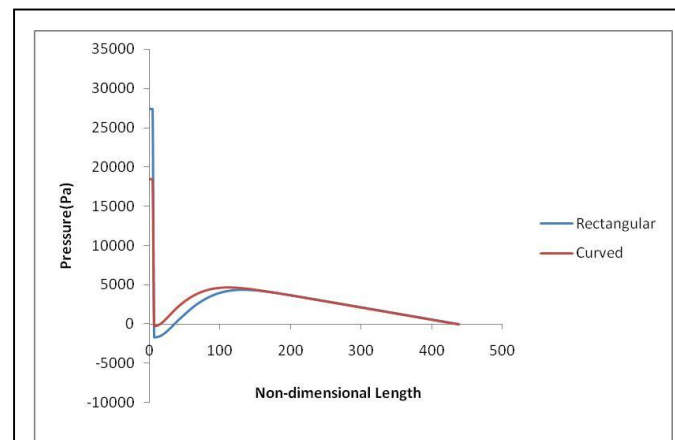
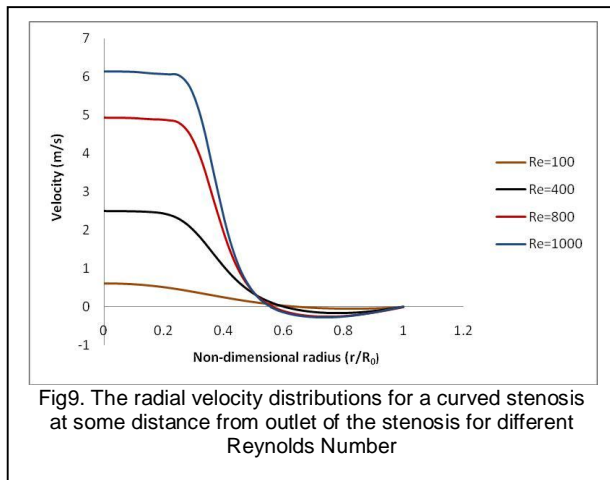


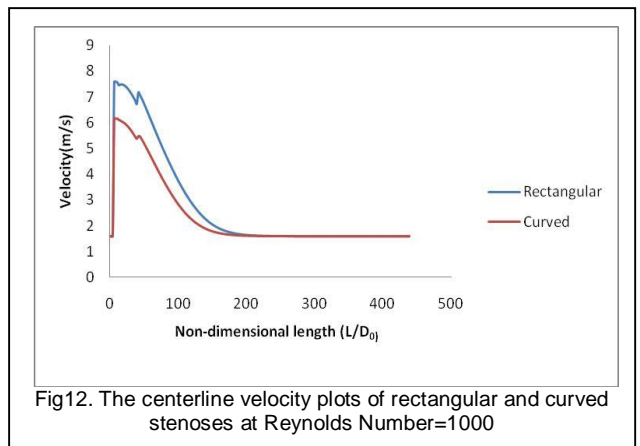
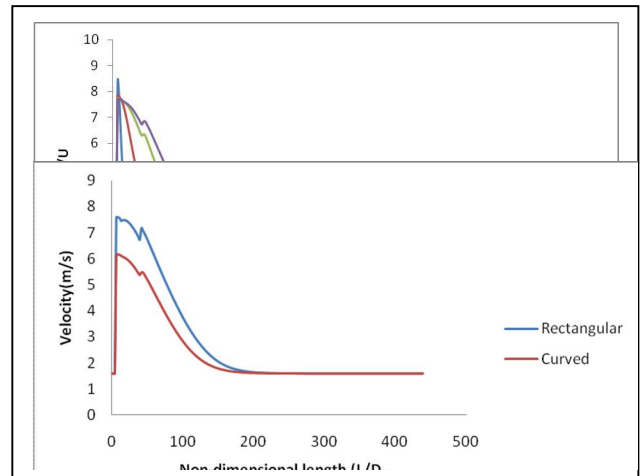
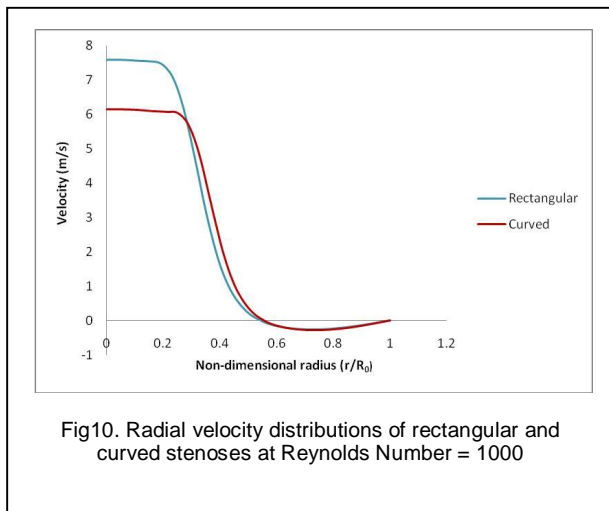
Fig8. Centerline pressure plots of Rectangular and curved stenosis at Reynolds Number=1000

### 4.2.2 Radial velocity field

As seen in Fig.9 the radial velocity field at the outlet of the stenosis shows negative velocity and the maximum value of the negative velocity is higher for higher values of Re. (For the rectangular stenosis, at the stenosis inlet, the maximum velocity has been found to shift from the centerline).



The maximum velocity in case of rectangular profile is higher than the curved geometry.



In our current study we are concentrating on two main single sided deposition geometry i.e. cosine shape and rectangular shape. But in our clinical study we not only get single deposition we encountered multiple depositions near about 20% cases.

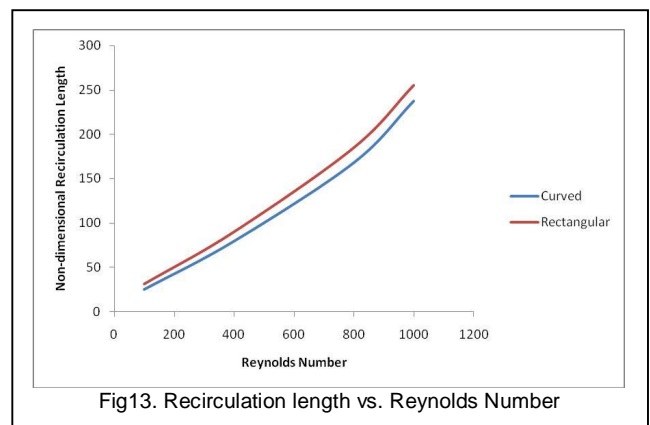
#### 4.2.3 Centerline velocity field

From the centerline velocity plot (Fig 11), the maximum velocity in the entire sub-domain has been found in a zone near the outlet of the stenosis for all the values of Re used. Also the length of reattachment of the flow after the flow separation has been found to increase with increasing values of Reynolds Number.

The reattachment length of the rectangular stenosis is around 10% more than the curved stenosis, as can be concluded from (Fig. 12).

#### 4.2.3 Centerline velocity field

The recirculation lengths of both the rectangular and curved stenoses plotted against the respective Reynolds Numbers show an almost linear variation. From this graph (Fig 13) it can be seen very clearly that the recirculation lengths of the rectangular stenosis is higher than a curved stenosis for the same value of Reynolds number.



The data set also reflects good amount of large sized plaque accompanied by smaller one or multiple depositions, where their dimension, placement, and appearance varies. Table 3 shows the distance between two adjacent



depositions, where almost no gap is considered for distance of < 1mm, small gap is considered for the distance between 1mm – 3mm, large gap is taken for the distance between 3mm – 6mm, and above this all the distances are considered as very large gap.

TABLE3  
DISTANCE BETWEEN MULTIPLE STENOSIS

Distance Between Multiple Deposition	Percentage of occurrence
Almost no gap between two adjacent formation	16.66 %
Small gap between two adjacent formation	50 %
Large gap between two adjacent deposition	25 %
Very Large gap between two adjacent deposition	8.33 %

From the clinical study it is found that multiple stenoses often occur in reality. It causes much more complexity in flow pattern. With such multiple depositions, size and shape of recirculation zone is expected to suffer variation. Numerical Modeling with such real life situations involving multiple stenosis is being done currently and will be reported later

## 5 INFERENCE

From the clinical study of more than 130 patients with carotid artery stenosis we come to the common inferences after detailed study of it, which are stated as follows.

More deposition is seen in ICA with respect to ECA proves the haemodynamic nature and construction of bifurcation geometry of human Carotid artery. A good amount of deposition is noted in the bulb and near bifurcation zone conforms the fact of flow separation and stagnation and deposition of substances in this type of regions.

Lower wall deposition is maximum for both single and both sided deposition stating stagnation of particles in the lower surface of a flowing liquid for a pipe flow.

From the geometry and appearance of multiple plaque and its internal distance it is clear that a single deposition initiate another deposition in its locality an along time this depositions merges to form a more complex and large structure which is clinic ally more dangerous.

In the present work the flow of blood through stenosed arteries has been studied by considering blood to be a Newtonian fluid and the flow to be laminar by varying the Reynolds Number.

For fixed stenosis geometry, as the pressure rise increases with increasing Reynolds Number, the heart has to supply even more pressure to overcome this adverse pressure gradient. Thus the effort of the heart increases, leading to angina (pain in the heart). Also as the recirculation zone is higher for higher values of the flow velocity, the tendency of the stenosis to propagate increases. This is because the stenosis aggravates with higher lengths of

re-attachment.

From the comparison of the rectangular and curved stenosis, it is inferred that as the extent of the recirculation length is longer in case of the rectangular stenosis than the curved one, the rectangular stenosis has a higher tendency to propagate. This is expected from the shapes of the stenoses. The rectangular stenosis presents itself as a bluff body in the line of flow of blood while the curved stenosis is more streamlined. Also that the irreversible pressure rise for the rectangular geometry is higher proves that it is more severe than a curved stenosis of the same maximum constriction due to causes already discussed.

It can also be predicted from the above study that as depositions continue to occur downstream of a curved stenosis, it will eventually develop into a rectangular stenosis if enough time is available. So, the adverse effects of a stenosis essentially increase with time.

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